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A UNIFIED PICTURE OF GLUEBALL CANDIDATES $f_0(1500)$ AND $f_0(1700)$

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ABSTRACT

A simple mixing scheme describing the $f_0(1500)$ and the $f_0(1700)$ as mixed states of a $\bar{s}s$ meson and a digluonium is reconsidered at the light of new experimental data.

In QCD, because of self interaction of gluons, one expects the existence of particles composed of gluons, which are called glueballs.

Many years have passed since the original prediction of the existence of these states [1], but, albeit the existence of many different candidates, up to now none of the many resonances has been assigned to a glueball.

Recently, two identifications of the lowest–lying scalar glueball have been proposed: one claim [2] is that it corresponds to a resonance observed at 1500 MeV (which can be identified with the former $f_0(1590)$, see [3]) that does not fit the usual meson nonet; the second [4] identifies this state with the resonance observed at 1700 MeV, which was known as $\Theta(1700)$.

Both the resonances have unusual decay properties for an ordinary $\bar{q}q$ meson, appear in gluon–rich channels and are in the mass region where lattice QCD predicts the existence of a scalar glueball [4,5]. Furthermore, for the $f_0(1700)$, Ref. [4] claims that the branching ratios are in agreement with a lattice QCD calculation and that the mass practically coincides with the predicted one.

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However, it must be emphasized that all available lattice calculations of the digluonium mass are made in the so–called quenched approximation, namely neglecting the creation of $\bar{q}q$ pairs: this casts some doubts about the reliability of these predictions, which can be considered only as indicative. Also, the prediction of branching ratios in lattice QCD has been questioned [6].

Anyway, it is quite puzzling to observe in the region where the scalar digluonium state is predicted two resonances not well fitting the usual meson nonets and with unusual decay properties.

In our opinion a common origin should be searched for both these states. Some years ago, a phenomenological scheme was proposed, consisting of a mixing between a glueball and a $\bar{s}s$ state [7]. The improved experimental data on these two resonances allow now quite a deeper investigation. In this paper this mixing scheme is reconsidered at the light of the new experimental inputs.

The mixing scheme is

$$\begin{aligned} \mid f_0(1500)\rangle &= \cos\alpha \mid gg\rangle - \sin\alpha \mid \bar{s}s\rangle \\ \mid f_0(1700)\rangle &= \sin\alpha \mid gg\rangle + \cos\alpha \mid \bar{s}s\rangle \end{aligned} \tag{1}$$

Given that the main $f_0(1700)$ decay is in the $\bar{K}K$ channel, it is assumed that the $\mid \bar{q}q\rangle$ ($\bar{u}u$ and $\bar{d}d$) component is negligible [7].

Of course, generally speaking, one could choose any arbitrary mixing between the scalar digluonium, the light quark $\mid \bar{q}q\rangle$ and the $\mid \bar{s}s\rangle$ states. At the moment the experimental data do not yet allow to exclude other choices, also because of the poor knowledge of the scalar nonet [8,9] compared to the other meson multiplets (it is worth to remember that strong instantonic effects are expected in the scalar sector [10]).

For example, quite a different perspective is adopted in [2], based on a mixing of the 0^{++} glueball with a light–quark system giving two physical states corresponding to the $f_0(1500)$ and $f_0(1400)$. In [11] the $f_0(1500)$ derives from a mixing of the $\mid \bar{q}q\rangle$ and the $\mid \bar{s}s\rangle$ states. Finally, in [12] an identification of $f_0(1500)$ with an unmixed glueball is suggested.

In the following, besides the mixing hypothesis (1), it is also assumed that the $f_0(1500)$ decouples from $\bar{K}K$, since no experimental evidence of this decay has been reported. This assumption permits to relate the decay amplitudes for the gg and the $\bar{s}s$ components through

$$\begin{aligned} &\langle K^+K^- \mid f_0(1500)\rangle = 0 = \\ &= \cos\alpha\langle K^+K^- \mid gg\rangle - \sin\alpha\langle K^+K^- \mid \bar{s}s\rangle \end{aligned} \tag{2}$$

Of course this is quite an approximation, but it is the simplest one and permits to obtain a large predictive power for the model. A small branching ratio of $f_0(1500) \to K\bar{K}$ or some different phase between the two amplitudes should not spoil the essence of the model. The agreement among the results and the experimental data confirms that this ansatz is acceptable.

To obtain the mixing angle from

$$\langle K^+K^- \mid f_0(1700)\rangle = \csc\alpha \langle K^+K^- \mid gg\rangle \tag{3}$$

one uses the isospin relations

$$\Gamma(f \to \pi\pi) = 3/2 \times \Gamma(f \to \pi^+\pi^-) \tag{4}$$

$$\Gamma(f \to KK) = 2 \times \Gamma(f \to K^+K^-); \tag{5}$$

requests $\langle \pi\pi \mid \bar{s}s\rangle = 0$ and assumes flavour independence

$$\langle K^+K^- \mid gg\rangle = \langle \pi^+\pi^- \mid gg\rangle.$$

Using the experimental values [3]

$$\begin{aligned} B.R.[f_0(1700) \to \pi\pi] &= 0.039^{+0.002}_{-0.024} \\ B.R.[f_0(1700) \to \bar{K}K] &= 0.38^{+0.09}_{-0.19} \end{aligned} \tag{6}$$

from Eq. (3), including the phase space factors p (meson momentum), one gets

$$\begin{aligned} \sin(\alpha) &= 0.579^{+0.072}_{-0.095} \\ \cos(\alpha) &= 0.815^{+0.067}_{-0.051} \end{aligned} \tag{7}$$

From the mixing angle α, one can immediately estimate the glueball and the scalar $\bar{s}s$ state masses [7], using the relations:

$$\begin{pmatrix} \bar{s}s & \epsilon \\ \epsilon & gg \end{pmatrix} \begin{pmatrix} -\sin\alpha \\ \cos\alpha \end{pmatrix} = 1.503 \begin{pmatrix} -\sin\alpha \\ \cos\alpha \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} \bar{s}s & \epsilon \\ \epsilon & gg \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} = 1.697 \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \tag{9}$$

where gg denotes the digluonium mass, $\bar{s}s$ the quark state mass and ϵ the mixing parameter.

This gives both $\bar{s}s$ and gg masses around 1.6 GeV, the latter in agrement with lattice predictions [5,4].

The mixing angle gives also access to many branching ratios. Using Ref. [2] and Eq. (3) one gets the following amplitudes relative to $\langle \pi\pi \mid gg\rangle$:

$$
\begin{aligned}
&\langle \pi\pi \mid \bar{s}s \rangle = 0 \\
&\langle K\bar{K} \mid \bar{s}s \rangle = R\cot(\alpha) \\
&\langle \eta\eta \mid \bar{s}s \rangle = 2R\sin^2(\phi)\cot(\alpha) \\
&\langle \eta\eta' \mid \bar{s}s \rangle = -2R\cos(\phi)\sin(\phi)\cot(\alpha) \\
&\langle K\bar{K} \mid gg \rangle = R \\
&\langle \eta\eta \mid gg \rangle = \cos^2(\phi) + R^2\sin^2(\phi) \\
&\langle \eta\eta' \mid gg \rangle = \cos(\phi)\sin(\phi)(1-R^2)\,,
\end{aligned}
\tag{10}
$$

where $R = \langle \bar{s}s \mid gg \rangle / \langle \bar{q}q \mid gg \rangle$ measurs the breaking of $SU_f(3)$ in gluonium decays (u and d quarks are assumed to be equivalent). No flavour violation is considered for the decay of quarkonium in pair of mesons (empirically this violation is shown to be quite small for the well–established meson nonets [13]).

In Eq. (10), ϕ is the angle for the $\eta - \eta'$ mixing

$$
\begin{aligned}
\eta &= \cos(\phi)|\bar{q}q\rangle - \sin(\phi)|\bar{s}s\rangle \\
\eta' &= \sin(\phi)|\bar{q}q\rangle + \cos(\phi)|\bar{s}s\rangle
\end{aligned}
\tag{11}
$$

In the following, a value $\phi = 72^o$ [14] is adopted; a possible gluonic component of the η and η' (see for example [15] and references therein) will not be considered here.

Furthermore, considering that glueballs should, at least in a first approximation, exhibit flavour democracy, flavour–independence for the gg decays (i.e. $R = 1$) as well is assumed.

Some predictions are listed here and compared with the data of Ref. [3]. The calculations include a sum over permutation and over the various charge combinations with the appropriate weighting factors (4 for $K\bar{K}$, 3 for $\pi\pi$, 2 for $\eta\eta'$ and 1 for $\eta\eta$). The theoretical errors only account for the uncertainty on the mixing angle.

$$
\frac{\Gamma(f_0(1700) \to \eta\eta)}{\Gamma(f_0(1700) \to \pi\pi)} = \frac{p_\eta}{3p_\pi} \cdot \left[1 + 2\sin^2(\phi)\cot^2(\alpha)\right]^2 = 5.43^{+3.0}_{-2.4} \tag{12}
$$

in agreement with the experimental datum $4.6^{+2.9}_{-3.3}$;

$$
\frac{\Gamma(f_0(1700) \to \eta\eta)}{\Gamma(f_0(1700) \to K\bar{K})} = \frac{p_\eta}{4p_K} \cdot \left[\sin^2(\alpha) + 2\sin^2(\phi)\cos^2(\alpha)\right]^2 = 0.558^{+0.080}_{-0.061} \tag{13}
$$

in good agreement with the datum $0.47^{+0.24}_{-0.35}$.

Similarly one finds:

$$
\frac{\Gamma(f_0(1500) \to \pi^0\pi^0)}{\Gamma(f_0(1500) \to \eta\eta)} = \frac{p_\pi}{p_\eta}\left[\frac{1}{\cos^2(\phi) - \sin^2(\phi)}\right]^2 = 2.2 \tag{14}
$$

in fair agreement with the two experimental data 1.45 ± 0.61, 2.12 ± 0.81.

A little more complicate is the evaluation of $\Gamma(f_0(1500) \to \eta\eta')/\Gamma(f_0(1500) \to \eta\eta)$ because $f_0(1500)$ just lies at the threshold for $\eta\eta'$ production. One has to consider a weighting with Breit-Wigner distribution through:

$$\frac{\Gamma(f_0(1500) \to \eta\eta')}{\Gamma(f_0(1500) \to \eta\eta)} =$$
$$2 \cdot \left[\frac{2\cos(\phi)\sin(\phi)}{\cos^2(\phi) - \sin^2(\phi)}\right]^2 \cdot \left[\frac{\int_{m_\eta + m_{\eta'}}^{\infty} dE/[(2(E-M_{f_0})/\Gamma)^2+1] \cdot \sqrt{\frac{(E^2 - m_\eta^2 - m_{\eta'}^2)^2 - 4m_\eta^2 m_{\eta'}^2}{4E^4}}}{\frac{p_\eta}{M_{f_0}} \int_0^{\infty} dE/[(2(E-M_{f_0})/\Gamma)^2+1]}\right]$$
$$= 0.232 \tag{15}$$

to be compared with the value 0.29 ± 0.10, which is the Crystal Barrel result reported in Ref. [3].

Analogously one has:

$$\frac{\Gamma(f_0(1700) \to \eta\eta')}{\Gamma(f_0(1700) \to K\bar{K})} =$$
$$1/2 \cdot \left[2\cos(\phi)\sin(\phi)\cos^2(\alpha)\right]^2 \cdot \left[\frac{\int_{m_\eta + m_{\eta'}}^{\infty} dE/[(2(E-M_{f_0})/\Gamma)^2+1] \cdot \sqrt{\frac{(E^2 - m_\eta^2 - m_{\eta'}^2)^2 - 4m_\eta^2 m_{\eta'}^2}{4E^4}}}{\frac{p_K}{M_{f_0}} \int_0^{\infty} dE/[(2(E-M_{f_0})/\Gamma)^2+1]}\right]$$
$$= 0.039^{+0.013}_{-0.010}, \tag{16}$$

to be compared with future experimental data.

Further predictions may be obtained considering the decay of J/Ψ (Υ) to $f_0(1500)$ and $f_0(1700)$ to proceed mainly through the gluonic component of these states. This assumption leads to:

$$\frac{\Gamma(J/\Psi \to \gamma f_0(1700))}{\Gamma(J/\Psi \to \gamma f_0(1500))} = \tan^2(\alpha) \cdot \left[\frac{M_{J/\Psi}^2 - 1.697^2}{M_{J/\Psi}^2 - 1.503^2}\right]^3 = 0.39^{+0.11}_{-0.14}, \tag{17}$$

$$\frac{\Gamma(\Upsilon \to \gamma f_0(1700))}{\Gamma(\Upsilon \to \gamma f_0(1500))} = 0.50^{+0.14}_{-0.18}, \tag{18}$$

At the moment only the branching ratios $B.R.[J/\Psi \to \gamma f_0(1700) \to \gamma K\bar{K})] = (9.7 \pm 1.2) \cdot 10^{-4}$ and $B.R.[J/\Psi \to \gamma f_0(1500) \to \gamma 4\pi)] = (8.2 \pm 1.5) \cdot 10^{-4}$ are available for the J/Ψ decays and only an upper limit for $f_0(1700)$ in the Υ case. Thus, due to the missing of the knowledge of $\Gamma(f_0(1500) \to 4\pi)/\Gamma(total)$, no real conclusion can still be drawn. When more stringent experimental results will be available the ratios (17,18) will represent a further test of this model.

Finally, another test of the model can be made considering the two–photon decays. Because gluons decouple from photons, this decay can proceed only through the quark component and is therefore partially suppressed for $f_0(1500)$ and $f_0(1700)$. In the mixing scheme (1), one expects

$$\frac{\Gamma(f_0(1700) \to \gamma\gamma)}{\Gamma(f_0(1500) \to \gamma\gamma)} = \left(\frac{1.697}{1.503}\right)^3 \cdot \cot^2(\alpha) = 2.85^{+1.0}_{-0.8} \tag{19}$$

If the $f_0(1400)$ is assumed to be the light quark isoscalar member of the 0^{++} nonet (but this assignation is still quite controversial [3,8]) one also predicts

$$\frac{\Gamma(f_0(1700) \to \gamma\gamma)}{\Gamma(f_0(1400) \to \gamma\gamma)} = 0.095^{+0.016}_{-0.012} \tag{20}$$

and

$$\frac{\Gamma(f_0(1500) \to \gamma\gamma)}{\Gamma(f_0(1400) \to \gamma\gamma)} = 0.033^{+0.008}_{-0.011} \tag{21}$$

In summary, it has been reconsidered a very simple mixing scheme which enables us to reproduce all the available experimental data (albeit still not very rich) on the two glueball candidates $f_0(1500)$ and $f_0(1700)$. In this scheme both resonances have a gluonic component mixed with a $\bar{s}s$ one.

The mixing angle has been obtained by a simple ansatz, allowing the evaluation of several ratios of branching ratios of the two resonances, in good agreement with the available experimental data.

Using the predictions reported in this paper it will be possible to test with a larger accuracy the model in a next future, when new experimental results will appear, permitting a clearer understanding of the nature of these two peculiar particles and hopefully a first certain identification of a glueball.

Clarifying the nature of these two resonances will be of great help for understanding of the composition of the 0^{++} meson nonet, which is still quite controversial [8,9].

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